# ANALYSIS OF THE PROCESS OF FILM CONDENSATION OF MOVING VAPOR ON A HORIZONTAL CYLINDER

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UDC 536.248.2

A method is presented for calculating the local and average heat transfer coefficients in film condensation of moving vapor of a nonmetallic fluid on a horizontal cylinder.

An analysis of the process of film condensation of moving vapor on the outer surface of a crossflow horizontal cylinder was the subject of study by a number of authors [2-9]. The solution of the problem for the case of stationary vapor is given in Nusselt's classical paper [1]. In [7, 8] the analysis of the process for the case of moving vapor is carried out with consideration of the effect of a directed mass flow through the interface on the hydrodynamics of the vapor—liquid boundary layer. However, the solution obtained in [7, 8] for the case of interest to us (for the case of the simultaneous action of friction and gravitation on the film) bears an approximate character and is in need of a more rigorous substantiation.

The analytical solution of the problem was given in [9] constructed on the physical model of the process proposed in [7, 8]. However, this solution was obtained by using certain simplifying assumptions whose legitimacy is not fully substantiated. In particular, as the authors themselves note [9], the approximation they use does not give sufficiently accurate results of the calculation for the afterbody of the cylinder. In addition, what is more important from the standpoint of use in practice, the results of calculating the average heat transfer coefficients are not given in [9] and there is no comparison with the equation of average heat transfer obtained in [7, 8]. All the foregoing also determined the need to obtain a new solution of the problem in question with the use of a more rigorous mathematical approach.

In analyzing the process (Fig. 1) the cross section of the cylinder is regarded as a regular polygon with a sufficiently large number of sides inscribed in a circle. The process of condensation on the cylinder in such a case reduces to condensation on a large number of sequentially arranged sides having different slope angles to the horizontal plane. The velocity of the vapor on the outer boundary of the vapor boundary layer and law of friction on the film surface are taken according to [7, 8]:

$$U_\varphi = 2U_\infty \sin\varphi, \tag{1}$$

$$\tau = \frac{q}{r\rho''} U_\varphi . \tag{2}$$

The resistance of the laminar film of condensate is taken as the main thermal resistance. We neglect the heat of supercooling of the condensate in comparison with the latent heat of condensation and the velocity at the interface in comparison with the vapor velocity at the outer boundary of the vapor boundary layer. In determining the geometry of the interface we neglect the thickness of the film in comparison with the diameter of the cylinder. We construct the solution for determining the local and average (over the cylinder's surface) heat transfer coefficient in the case of flow without separation at a constant surface temperature of the cylinder. To determine the average coefficient we find the average heat transfer coefficient on the first (starting from the frontal point) side and the local heat transfer coefficients on the middle generators of the other sides of the cylinder, which in connection with the sufficiently large value of the number n are equated to the average heat transfer coefficients of the sides. The average heat transfer coefficient being sought is defined as the arithmetic mean value of the indicated quantities (in view of symmetry it is sufficient to consider only on half of the cylinder). To calculate the required quantities we

Scientific-Research Institute of Stable Isotopes, Tbilisi. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 25, No. 1, pp. 14-19, July, 1973. Original article submitted January 8, 1973.

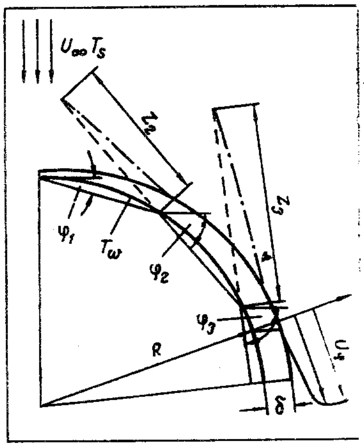


Fig. 1. Scheme of analyzing the process.

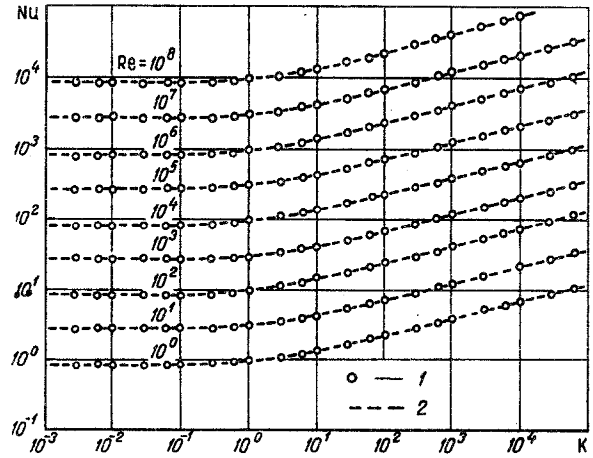


Fig. 2. Comparison of the results of the given analysis with Eq. (13): 1) calculated values; 2) Eq. (13).

generalize the solution of the problem of moving vapor on a vertical plate obtained in [8] for the case of any slope in a gravitational field. For the local coefficient on the side of the cylinder with slope $\varphi_n$ with consideration of the adopted law of distribution of the vapor velocity on the outer boundary of vapor boundary layer we obtain

$$\alpha = \sqrt{\frac{\lambda^2 \rho U_\infty \sin\varphi_n}{4\mu x_n}} \sqrt{1 + \sqrt{1 + \frac{4 g x_n}{N U_\infty^2 \sin\varphi_n}}}, \tag{3}$$

where the dimensionless complex

$$N = \frac{\lambda \Delta T}{r\mu}. \tag{4}$$

The average heat transfer coefficient on the first side of the cylinder

$$\bar{\alpha}_1 = \frac{1}{3}\sqrt{\frac{4\lambda^2 \rho U_\infty \sin\varphi_1}{\mu l}} \cdot \frac{2 + \sqrt{1 + \dfrac{4gl}{N U_\infty^2 \sin\varphi_1}}}{\sqrt{1 + \sqrt{1 + \dfrac{4gl}{N U_\infty^2 \sin\varphi_1}}}}. \tag{5}$$

To find the required quantities for the other sides of the cylinder by Eq. (3), for each side we must determine the presently unknown lengths $X_n$. For the middle generators of the sides of the cylinder these lengths are equal to

$$X_n = Z_n + \frac{l}{2}. \tag{6}$$

It is obvious physically that for a correct determination of local heat transfer by Eq. (3) the length $Z_n$ should be determined from the condition of equality of the quantities of condensate that condensed on length $Z_n$ on one hand and on all preceding sides of the cylinder on the other. This condition is written so:

$$\bar{\alpha}_{zn}(T_s - T_w) Z_n = (\bar{\alpha}_1 + \alpha_2 + \ldots + \alpha_{n-1})(T_s - T_w) l. \tag{7}$$

The search for $X_n$ according to condition (6) is done successively for each subsequent side and leads to the following calculation model:

$$Z_1 = 0, \tag{8}$$

$$Z_n = \left[-\left(\frac{1}{B_n^3} + \frac{5C_{n-1}^2}{A_n^2 B_n^2} - \frac{C_{n-1}^4}{2A_n^4 B_n}\right) + \left[\frac{C_{n-1}^8}{4A_n^8 B_n^2} + \frac{3C_{n-1}^6}{A_n^6 B_n^3} + \frac{12C_{n-1}^4}{A_n^4 B_n^4} + \frac{16C_{n-1}^2}{A_n^2 B_n^5}\right]^{1/2}\right]^{1/3}$$
$$+ \left[-\left(\frac{1}{B_n^3} + \frac{5C_{n-1}^2}{A_n^2 B_n^2} - \frac{C_{n-1}^4}{2A_n^4 B_n}\right) - \left[\frac{C_{n-1}^8}{4A_n^8 B_n^2} + \frac{3C_{n-1}^6}{A_n^6 B_n^3} + \frac{12C_{n-1}^4}{A_n^4 B_n^4} + \frac{16C_{n-1}^2}{A_n^2 B_n^5}\right]^{1/2}\right]^{1/3} + \frac{2}{B_n}, \tag{9}$$

where

$$A_n = \frac{1}{3}\sqrt{\frac{4\lambda^2 \rho U_\infty \sin\varphi_n}{\mu}}; \tag{10}$$

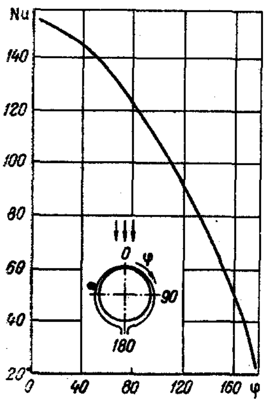

Fig. 3

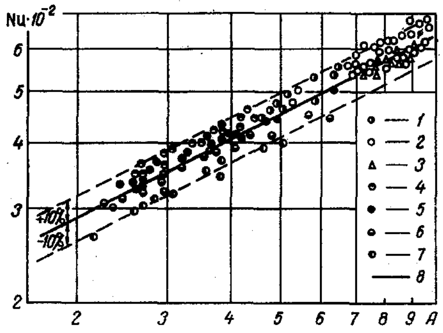

Fig. 4

Fig. 3. Distribution of local Nusselt number over the circumference of the cylinder. Re = $10^4$; K = 6.

Fig. 4. Comparison of the experimental data of [5, 10] with Eq. (13): 1) $P = 4.7\ N/cm^2$; $\Delta T = 7.4°C$ [5]; 2) $P = 4.7\ N/cm^2$; $\Delta T = 2.5°C$ [5]; 3) $P = 0.31\ N/cm^2$; $\Delta T = 1.2°C$ [5]; 4) $U_\infty = 0.56$ m/sec [10]; 5) $U_\infty = 0.37$ m/sec [10]; 6) $U_\infty = 0.22$ m/sec [10]; 7) $U_\infty = 0.11$ m/sec [10]; 8) Eq. (13). $A = Re[1 + \sqrt{1 + 1.69K}] \cdot 10^{-5}$.

$$B_n = \frac{4g}{NU_\infty^2 \sin\varphi_n}; \quad (11)$$

$$C_n = (\bar{\alpha}_1 + \alpha_2 + \ldots + \alpha_n)\, l. \quad (12)$$

It is easy to note that the calculations of the local and average heat transfer coefficients by the solution obtained are cumbersome and in the case of noncomputer calculation require a quite long time. In this connection we made the appropriate calculations on the M-220 computer of the Institute of Applied Mathematics and Mechanics, Tbilisi State University. We calculated the values of the local and average (for the cylinder) Nusselt numbers $\overline{Nu} = \bar{\alpha}D/\lambda$ in a large number of variants as a function of the dimensionless parameters $Re = U_\infty D/\nu$ and $K = gDr\mu/\lambda\Delta T U_\infty^2$. We covered practically the entire range of variation of these parameters of interest from the viewpoint of engineering ($1 \le Re \le 10^6$; $10^{-5} \le K \le 10^5$). The acceptable value of the number n was determined before making the final calculations. As the convergence of the results for values of this number equal to 9, 18, 36, and 72 showed, with an increase of this number, beginning with 18, the results of the calculations practically do not change. In this connection, taking into the account the opportunities offered by the computer, we made the final calculations for n = 72.

The results of the calculations by the given solution were compared with the calculations by the approximate Eq. (28) in [8], having in dimensionless notation the following form:

$$\overline{Nu} = 0.64\sqrt{Re}\sqrt{1 + \sqrt{1 + 1.69K}}. \quad (13)$$

As we see from Fig. 2, the results of the given calculation with respect to average heat transfer coefficients coincide practically completely with (13) obtained for the case of vapor flow past a cylinder without separation. The indicated result can be regarded as confirmation of the legitimacy of the assumptions underlying Eq. (13). It is obvious that it is vastly more convenient to make practical calculations of the average heat transfer by this relationship, which is a good approximation of the results of the given analysis.

Figure 3 shows the distribution of the local Nusselt number over the circumference of the cylinder obtained for the case characterized by an approximately equal effect of the interfacial friction and gravitational field on the process. It is interesting to note that in calculating the local heat transfer by the above-indicated step at the frontal zone, a local minimum is observed on the heat transfer curve. However, in calculating with a smaller step this minimum is eliminated. This is explained by the fact that in the neighborhood of the frontal point the multiple step, equal to 5°, is nevertheless great and leads to sufficiently marked changes of the longitudinal forces of friction and gravity, which in turn leads to artificial disruption of the monotonicity of the dependence of the heat transfer coefficient on angle $\varphi$. The indicated minimum has practically no effect on the values of the local heat transfer coefficient, beginning with angle $\varphi$ of about 15°, and average heat transfer coefficient for the entire cylinder.

In Fig. 4 Eq. (13) is compared with the literature data on condensation of moving vapor on a horizontal cylinder [5, 10] under conditions of such oncoming flow velocities that separation of the vapor boundary layer from the surface of the cylinder is improbable. The average film temperature was used as the design temperature for the parameters of the condensate in the comparison. The physical properties of Freon-21 were taken from handbooks [11, 12].

As we see from Fig. 4, Eq. (13) satisfactorily generalizes the experimental data obtained on condensation of vapors of two different liquids.*

## NOTATION

| | |
|---|---|
| $\varphi$ | is the angle reckoned from the frontal point of the cylinder; |
| $U_\infty$, $U_\varphi$ | are the vapor velocities at infinity from the cylinder and on the outer boundary of the vapor boundary layer; |
| $T_s$, $T_w$ | are the temperatures of vapor saturation and surface of the cylinder; |
| P | is the pressure; |
| $\tau$ | is the interfacial shear stress; |
| Nu | is the Nusselt number; |
| Re | is the modified Reynolds number; |
| N, K | are the dimensionless complexes; |
| q | is the heat flux; |
| $\alpha$ | is the heat transfer coefficient; |
| $\lambda$ | is the thermal conductivity of liquid; |
| $\rho$ | is the density of liquid; |
| $\rho''$ | is the density of vapor; |
| $\mu$, $\nu$ | are the dynamic and kinematic viscosity; |
| r | is the latent heat of condensation; |
| g | is the acceleration of gravity; |
| D, R | are the diameter and radius of cylinder; |
| $l$ | is the length of side of inscribed polygon; |
| $Z_n$ | is the length of conditional equivalent plate. |

### Subscripts

| | |
|---|---|
| 1, 2, . . . , n | denotes the side of the inscribed polygon; |
| zn | denotes for a plate with length $Z_n$ and slope $\varphi_n$. |

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* In [10] the curves according to Eq. (13) are plotted with a certain error, which led to an increase of the divergence between theory and experiment by as much as 20%.